647510 **0301207C1010** **Seat No: ____________**

B.Sc. Semester - 2 (CBCS) Examination

March/April- 2019 (Old Course)

MATHEMATICS

(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1 (A) Answer the followings questions briefly: **(04)**

(1)Define: Right Circular cylinder.

(2) Find the sphere for which $A(1,2,3)$ and $B(4,3,2)$ are extremities of a diameter.

(3) Write the equation of cylinder whose axis is parallel to Y-axis and radius r.

(4) Write the equation of sphere in vector form.

(B) Answer any one. **(02)**

(1) Find the equation of the sphere through the circle
$x^2+y^2+z^2=9, 2x+3y+4z=5$and the point $(1,2,3)$.

(2) Show that the sphere with centre $(7,14,7)$ and radius 13 touches the plane $3x+12y+4z=48$.

(C) Answer any one. **(03)**

(1) Find the co-ordinates of the points where the line
$\frac{x-1}{2}=\frac{y-6}{3}=\frac{z-4}{4}$ intersects the sphere $x^2+y^2+z^2=10$.

(2) Find equation of right circular cylinder passing through $(1,-2,3)$and axis is $\{(2,-3,5+k)/k\in R\}$.

(D) Answer any one. **(05)**

(1) Derive equation of a cylinder of which generator remain parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and passing through a guiding curve $ax^2+2hxy+by^2+2gx+2fy+c=0; z=0$.

(2) Find out the equation of tangent plane to the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \text{at } (\alpha,\beta,\gamma).$$

Que-2 (A) Answer the followings questions briefly: **(04)**

(1) Evaluate: $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$

(2) Define: partial derivatives of function of two variables.

(3) If $z=y^x$ then find $\frac{\partial z}{\partial y}$.

(4) Define: Rectangular neighborhood.

(B) Answer any one. **(02)**

(1) Find first order partial derivatives of $f(x,y)=x\tan\frac{y}{x}$ at $(4,\pi)$.

(2) If $f(x,y)=\frac{xy}{\sqrt{x^2+y^2}}; (x,y)\neq(0,0)$
$= 0 \quad ; (x,y)=(0,0)$
then discuss the continuity at $(0,0)$.

(C) Answer any one. **(03)**

(1) If $u=f(z)$ and $z^2=x^2+y^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(z)+\frac{1}{z}f'(z).$$

(2) If $f(x,y)=\sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$ then show that $\frac{f_x}{f_y}=\frac{-y}{x}$.

(D) Answer any one. **(05)**

(1) State and prove Euler's theorem for homogeneous function of two variables.

(2) If $u=f(x,y)$ and $x=r\cos\theta, y=r\sin\theta$ then prove that:

(i) $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$

(ii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$.

Que-3 (A) Answer the followings questions briefly: **(04)**

(1) Define: local maxima and minima.

(2) If $x = r\cos\theta, y = r\sin\theta$ then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

(3) Define: extreme point.

(4) Find critical point of function $f(x,y) = x^2 + 2y^2 - x$.

(B) Answer any one. **(02)**

(1) If there is 0.05% error obtain in measurement of length of sides of rectangle then what should be error in the measurement of its area.

(2) find maxima and minima of function $f(x,y) = x^3 + y^3 - 3xy$.

(C) Answer any one. **(03)**

(1) Expand $e^x\cos y$ in powers of x and y up to three degree.

(2) If $f(x,y) = x^2y - 3y$ then find the approximate value of $f(5.12, 6.85)$.

(D) Answer any one. **(05)**

(1) State and prove Taylor's theorem.

(2) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Que-4 (A) Answer the followings questions briefly: **(04)**

(1) Define: Idempotent matrix.

(2) Show that $\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ is Nilpotent matrix of index 2.

(3) Define: Rank of a matrix.

(4) Give an example of symmetric matrix.

(B) Answer any one. **(02)**

(1) If $AB = BA$and $S^2 = B$ then show that $(A^{-1}SA)^2 = B$.

(2) Show that $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal.

(C) Answer any one. **(03)**

(1) Find adjoint of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(2) Find the rank of matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ by represents it in to normal form.

(D) Answer any one. **(05)**

(1) Show that every square matrix is uniquely expressible as the sum of a Hermitian and a skew-Hermition matrix.

(2) Define: Adjoint of a matrix and if $A = [a_{ij}]_n, B = [b_{ij}]_n$

then prove that: $A \cdot (adjA) = (adjA) \cdot A = |A|I$

Que-5(A) Answer the followings questions briefly: **(04)**

(1) Define: Characteristic equation.

(2) Find the characteristic equation of $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(3)The equation $AX = 0$ has nonzero solution if $A =$

(4) Define: Augmented matrix.

(B) Answer any one. **(02)**

(1) Prove that 0 is a characteristic root of a matrix iff matrix is singular.

(2) Find the eigen values of $\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(C) Answer any one. **(03)**

(1) Solve: $5x + 2y + 2z = 17, 9x + 4y + 2z = 27$.

(2) Prove that eigen values of Hermitian matrix is real numbers.

(D) Answer any one. **(05)**

(1) State and prove Cayley-Hamilton theorem.

(2) Solve equations:

$$x + y + z + w = 0, x + 3y + 2z + 4w = 0, 2x + z - w = 0.$$

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